AP UNIT: Polynomial Equations and Inequalities

Review of prerequisite skills
Student should be familiar with the following skills:

- **function notation**
- **algebraic simplification**
- **different types of factoring(common factoring, factoring trinomials, difference of squares, factoring by grouping, variable substitution technique and combinations of aforementioned)**
- **solve by factoring**
- **solve by using quadratic formula**

Upon completion of this package, student should be able to:

- **Polynomial Division -** Long Division and Synthetic
- Make connections between a polynomial function, f(x), the divisor (x –a), and the remainder from the division $\frac{f(x)}{(x-a)}$ and verify **the remainder theorem** and **factor theorem.**
- **Factor polynomial expressions in one variable** of degree no higher than 4 by selecting and applying strategies.
- Determining the connection between the real roots of a polynomial equation and the x-intercepts of the graph of the corresponding polynomial function and describe this connection.
- **Solve polynomial equations in one variable**, of degree no higher than 4 by selecting and applying strategies.
- Explain, for polynomial functions, the difference between the solution to an equation in one variable and the solution to an inequality in one variable and demonstrate that the given solutions satisfy an inequality.
- **Characteristics of Polynomial Functions –** degree, leading coefficient, end behaviour, turning points, zeroes, order.
  http://tutorial.math.lamar.edu/Classes/Alg/GraphingPolynomials.aspx
- **Determine solutions to polynomial inequalities in one variable by graphing** the corresponding functions and identifying intervals for which x satisfies the inequalities.
- **Solve linear inequalities and factoring polynomial inequalities** in a variety of ways and represent the solutions on a number line or algebraically.

Student Resources:

- *http://tutorial.math.lamar.edu/Classes/Alg/Alg.aspx*
- *www.classzone.com*
- *www.purplemath.com*
- *www.mathforum.org*
- *www.mathworld.wolfram.com*
- *www.math.com*
- *Use above highlighted words to search YOU TUBE lessons*

1.

Prerequisite Skills

Use Long Division

1. Use long division to find each quotient. Write the remainder.
   a) $3476 \div 28$
   b) $5973 \div 37$
   c) $2508 \div 17$
   d) $6815 \div 19$

Evaluate Functions

2. Given $P(x) = x^3 - 5x^2 + 7x - 9$, evaluate.
   a) $P(-1)$
   b) $P(3)$
   c) $P(-2)$
   d) $P\left(-\frac{1}{2}\right)$
   e) $P\left(\frac{2}{3}\right)$

Simplify Expressions

3. Expand and simplify.
   a) $(x^3 + 3x^2 - x + 1)(x - 2) + 5$
   b) $(2x^3 - 4x^2 + x - 3)(x + 4) - 7$
   c) $(x^3 + 4x^2 - x + 8)(3x - 1) + 6$
   d) $(x - \sqrt{2})(x + \sqrt{2})$
   e) $(x - 3\sqrt{5})(x + 3\sqrt{5})$
   f) $(x - 1 + \sqrt{3})(x - 1 - \sqrt{3})$

Factor Expressions

4. Factor each difference of squares. Look for common factors first.
   a) $x^2 - 4$
   b) $25m^2 - 49$
   c) $16y^2 - 9$
   d) $12c^2 - 27$
   e) $2x^4 - 32$
   f) $3n^4 - 12$

5. Factor each trinomial.
   a) $x^2 + 5x + 6$
   b) $x^2 - 9x + 20$
   c) $b^2 + 5b - 14$
   d) $2x^2 - 7x - 15$
   e) $4x^2 - 12x + 9$
   f) $6a^2 - 7a + 2$
   g) $9m^2 - 24m + 16$
   h) $3m^2 - 10m + 3$

Solve Quadratic Equations

6. Solve by factoring.
   a) $x^2 - 2x - 15 = 0$
   b) $4x^2 + x - 3 = 0$
   c) $16x^2 - 36 = 0$
   d) $9x^2 = -15 + 48x$
   e) $20 - 12x = 8x^2$
   f) $21x^2 + 1 = 10x$

7. Use the quadratic formula to solve. Round answers to one decimal place.
   a) $5x^2 + 6x - 1 = 0$
   b) $2x^2 - 7x + 4 = 0$
   c) $4x^2 = -2x + 3$
   d) $7x + 20 = 6x^2$

Determine Equations of Quadratic Functions

8. Determine an equation for the quadratic function, with the given zeros, and that passes through the given point.
   a) zeros: -4 and 1; point: $(-1, 2)$
   b) zeros: 0 and 3; point: $(2, 6)$
   c) zeros: -3 and 4; point: $(3, 24)$
   d) zeros: 5 and -1; point: $(4, -10)$
   e) zeros: $\frac{3}{2}$ and $-\frac{1}{2}$; point: $(0, 9)$

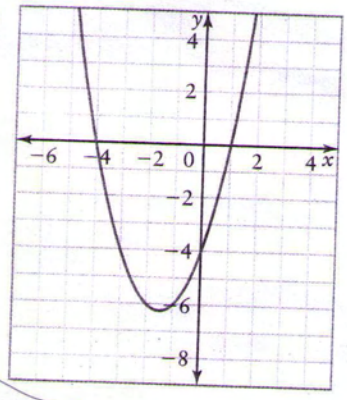
Determine Intervals From Graphs

9. For the graph of each polynomial function,

i) identify the x-intercepts

ii) write the intervals for which the graph is above the x-axis and the intervals for which the graph is below the x-axis

a)



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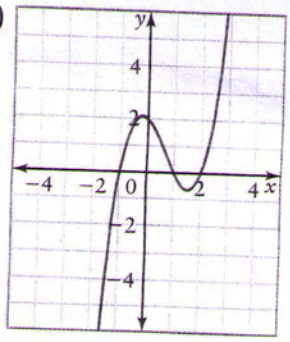
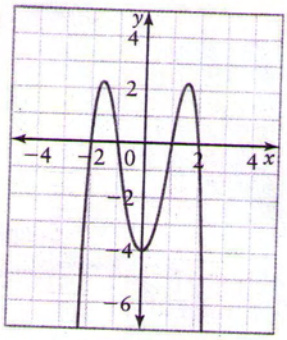


c)



Solutions: Prerequisite skills

Prerequisite Skills,

1. a) 124 R4 **b)** 161 R16 **c)** 147 R9 **d)** 358 R13

2. a) -22 **b)** -6 **c)** -51 **d)** -13.875 **e)** $-\frac{169}{27}$

3. a) $x^4 + x^3 - 7x^2 + 3x + 3$

b) $2x^4 + 4x^3 - 15x^2 + x - 19$

c) $3x^4 + 11x^3 - 7x^2 + 25x - 2$

d) $x^2 - 2$ **e)** $x^2 - 45$ **f)** $x^2 - 2x - 2$

4. a) $(x - 2)(x + 2)$ **b)** $(5m - 7)(5m + 7)$

c) $(4y - 3)(4y + 3)$ **d)** $3(2c - 3)(2c + 3)$

e) $2(x - 2)(x + 2)(x^2 + 4)$ **f)** $3(n^2 - 2)(n^2 + 2)$

5. a) $(x + 3)(x + 2)$ **b)** $(x - 4)(x - 5)$

c) $(b + 7)(b - 2)$ **d)** $(2x + 3)(x - 5)$

e) $(2x - 3)^2$ **f)** $(2a - 1)(3a - 2)$

g) $(3m - 4)^2$ **h)** $(m - 3)(3m - 1)$

6. a) $x = -3$ or $x = 5$ **b)** $x = -1$ or $x = \frac{3}{4}$

c) $x = -\frac{3}{2}$ or $x = \frac{3}{2}$ **d)** $x = \frac{1}{3}$ or $x = 5$

e) $x = -\frac{5}{2}$ or $x = 1$ **f)** $x = \frac{1}{7}$ or $x = \frac{1}{3}$

7. a) $x \doteq -1.3$ or $x \doteq 0.1$

b) $x \doteq 0.7$ or $x \doteq 2.8$

c) $x \doteq -1.2$ or $x \doteq 0.7$

d) $x \doteq -1.3$ or $x = 2.5$

8. a) $y = -\frac{1}{3}(x + 4)(x - 1)$

b) $y = -3x(x - 3)$ **c)** $y = -4(x + 3)(x - 4)$

d) $y = 2(x + 1)(x - 5)$ **e)** $y = -3(2x + 1)(2x - 3)$

9. a) i) -4 and 1 **ii)** above the x-axis: $x < -4$ and $x > 1$; below the x-axis: $-4 < x < 1$ **b) i)** -1, 1, and 2 **ii)** above the x-axis: $-1 < x < 1$ and $x > 2$; below the x-axis: $x < -1$ and $1 < x < 2$ **c) i)** -2, -1, 1, and 2 **ii)** above the x-axis: $-2 < x < -1$ and $1 < x < 2$; below the x-axis: $x < -2$ and $-1 < x < 1$ and $x > 2$

2. The Remainder Theorem

1. a) Divide $x^3 + 3x^2 - 2x + 5$ by $x + 1$. Express the result in quotient form.
b) Identify any restrictions on the variable.
c) Write the corresponding statement that can be used to check the division.
d) Verify your answer.

2. a) Divide $3x^4 - 4x^3 - 6x^2 + 17x - 8$ by $3x - 4$. Express the result in quotient form.
b) Identify any restrictions on the variable.
c) Write the corresponding statement that can be used to check the division.
d) Verify your answer.

3. Perform each division. Express the result in quotient form. Identify any restrictions on the variable.
a) $x^3 + 7x^2 - 3x + 4$ divided by $x + 2$
b) $6x^3 + x^2 - 14x - 6$ divided by $3x + 2$
c) $10x^3 + 11 - 9x^2 - 8x$ divided by $5x - 2$
d) $11x - 4x^4 - 7$ divided by $x - 3$
e) $3 + x^2 + 7x + 6x^3$ divided by $3x + 2$
f) $8x^3 + 4x^2 - 31$ divided by $2x - 3$
g) $6x^2 - 6 + 8x^3$ divided by $4x - 3$

4. Determine the remainder R so that each statement is true.
a) $(2x - 3)(3x + 4) + R = 6x^2 - x + 15$
b) $(x + 2)(x^2 - 3x + 4) + R = x^3 - x^2 - 2x - 1$
c) $(x - 4)(2x^2 + 3x - 1) + R = 2x^3 - 5x^2 - 13x + 2$

5. The volume, in cubic centimetres, of a rectangular box can be modelled by the polynomial expression $2x^3 + 17x^2 + 38x + 15$. Determine possible dimensions of the box if the height, in centimetres, is given by $x + 5$.

6. The volume, in cubic centimetres, of a square-based box is given by $9x^3 + 24x^2 - 44x + 16$. Determine possible dimensions of the box if the area of the base, in square centimetres, is $9x^2 - 12x + 4$.

7. Use the remainder theorem to determine the remainder when $2x^3 + 7x^2 - 8x + 3$ is divided by each binomial. Verify your answer using long division.
a) $x + 1$ b) $x - 2$ c) $x + 3$
d) $x - 4$ e) $x - 1$

8. Determine the remainder when each polynomial is divided by $x + 2$.
a) $x^3 + 3x^2 - 5x + 2$
b) $2x^3 - x^2 - 3x + 1$
c) $x^4 + x^3 - 5x^2 + 2x - 7$

9. Use the remainder theorem to determine the remainder for each division.
a) $x^3 + 2x^2 - 3x + 9$ divided by $x + 3$
b) $2x^3 + 7x^2 - x + 1$ divided by $x + 2$
c) $x^3 + 2x^2 - 3x + 5$ divided by $x - 3$
d) $x^4 - 3x^2 - 5x + 2$ divided by $x - 2$

SOLUTIONS

The Remainder Theorem,

1. a) $\frac{x^3 + 3x^2 - 2x + 5}{x + 1} = x^2 + 2x - 4 + \frac{9}{x + 1}$ b) $x \neq -1$
c) $(x + 1)(x^2 + 2x - 4) + 9$

2. a) $\frac{3x^4 - 4x^3 - 6x^2 + 17x - 8}{3x - 4} = x^3 - 2x + 3 + \frac{4}{3x - 4}$
b) $x \neq \frac{4}{3}$
c) $3x^4 - 4x^3 - 6x^2 + 17x - 8 = (3x - 4)(x^3 - 2x + 3) + 4$

3. a) $\frac{x^3 + 7x^2 - 3x + 4}{x + 2} = x^2 + 5x - 13 + \frac{30}{x + 2}, x \neq -2$
b) $\frac{6x^3 + x^2 - 14x - 6}{3x + 2} = 2x^2 - x - 4 + \frac{2}{3x + 2}, x \neq -\frac{2}{3}$
c) $\frac{10x^3 - 9x^2 - 8x + 11}{5x - 2} = 2x^2 - x - 2 + \frac{7}{5x - 2}, x \neq \frac{2}{5}$
d) $\frac{-4x^4 + 11x - 7}{x - 3} = -4x^3 - 12x^2 - 36x - 97 - \frac{298}{x - 3}, x \neq 3$
e) $\frac{6x^3 + x^2 + 7x + 3}{3x + 2} = 2x^2 - x + 3 - \frac{3}{3x + 2}, x \neq -\frac{2}{3}$
f) $\frac{8x^3 + 4x^2 - 31}{2x - 3} = 4x^2 + 8x + 12 + \frac{5}{2x - 3}, x \neq \frac{3}{2}$
g) $\frac{8x^3 + 6x^2 - 6}{4x - 3} = 2x^2 + 3x + \frac{9}{4} + \frac{3}{4(4x - 3)}, x \neq \frac{3}{4}$

4. a) 27 b) -9 c) -2
5. $(x + 5)(x + 3)(2x + 1)$
6. $(3x - 2)$ cm by $(3x - 2)$ cm by $(x + 4)$ cm
7. a) 16 b) 31 c) 36 d) 211 e) 4
8. a) 16 b) -13 c) -23
9. a) 9 b) 15 c) 41 d) -4

3. The Factor Theorem

1. Write the binomial factor that corresponds to the polynomial $P(x)$.

a) $P(4) = 0$ **b)** $P(-3) = 0$

c) $P\left(\frac{2}{3}\right) = 0$ **d)** $P\left(-\frac{1}{4}\right) = 0$

2. Determine if $x + 3$ is a factor of each polynomial.

a) $x^3 + x^2 - x + 6$

b) $2x^3 + 9x^2 + 10x + 3$

c) $x^3 + 27$

3. List the values that could be zeros of each polynomial. Then, factor the polynomial.

a) $x^3 + 3x^2 - 6x - 8$

b) $x^3 + 4x^2 - 15x - 18$

c) $x^3 - 3x^2 - 10x + 24$

4. Factor each polynomial by grouping terms.

a) $x^3 + x^2 - 9x - 9$

b) $x^3 - x^2 - 16x + 16$

c) $2x^3 - x^2 - 72x + 36$

d) $x^3 - 7x^2 - 4x + 28$

e) $3x^3 + 2x^2 - 75x - 50$

f) $2x^4 + 3x^3 - 32x^2 - 48x$

5. Determine the values that could be zeros of each polynomial. Then, factor the polynomial.

a) $3x^3 + x^2 - 22x - 24$

b) $2x^3 - 9x^2 + 10x - 3$

c) $6x^3 - 11x^2 - 26x + 15$

d) $4x^3 + 3x^2 - 4x - 3$

6. Factor each polynomial.

a) $x^3 + 2x^2 - x - 2$

b) $x^3 + 4x^2 - 7x - 10$

c) $x^3 - 5x^2 - 4x + 20$

d) $x^3 + 5x^2 + 3x - 4$

e) $x^3 - 4x^2 - 11x + 30$

f) $x^4 - 4x^3 - x^2 + 16x - 12$

g) $x^4 - 2x^3 - 13x^2 + 14x + 24$

11. Factor each polynomial.

a) $2x^3 + 5x^2 - x - 6$

b) $4x^3 - 7x - 3$

c) $6x^3 + 5x^2 - 21x + 10$

d) $4x^3 - 8x^2 + 3x - 6$

e) $2x^3 + x^2 + x - 1$

f) $x^4 - 15x^2 - 10x + 24$

Solutions

The Factor Theorem,

1. a) $x - 4$ **b)** $x + 3$ **c)** $3x - 2$ **d)** $4x + 1$
2. a) No. **b)** Yes. **c)** Yes.
3. a) $(x - 2)(x + 1)(x + 4)$ **b)** $(x - 3)(x + 1)(x + 6)$
c) $(x - 4)(x - 2)(x + 3)$
4. a) $(x - 3)(x + 1)(x + 3)$ **b)** $(x - 4)(x - 1)(x + 4)$
c) $(x - 6)(x + 6)(2x - 1)$ **d)** $(x - 7)(x - 2)(x + 2)$
e) $(x - 5)(x + 5)(3x + 2)$ **f)** $x(x - 4)(x + 4)(2x + 3)$
5. a) $(x - 3)(x + 2)(3x + 4)$ **b)** $(x - 3)(x - 1)(2x - 1)$
c) $(x - 3)(2x - 1)(3x + 5)$ **d)** $(x - 1)(x + 1)(4x + 3)$
6. a) $(x - 1)(x + 1)(x + 2)$ **b)** $(x - 2)(x + 1)(x + 5)$
c) $(x - 5)(x - 2)(x + 2)$ **d)** $(x + 4)(x^2 + x - 1)$
e) $(x - 5)(x - 2)(x + 3)$ **f)** $(x - 3)(x + 2)(x - 1)(x - 2)$
g) $(x - 4)(x - 2)(x + 1)(x + 3)$

11. a) $(x - 1)(x + 2)(2x + 3)$ **b)** $(x + 1)(2x - 3)(2x + 1)$
c) $(x - 1)(2x + 5)(3x - 2)$ **d)** $(x - 2)(4x^2 + 3)$
e) $(2x - 1)(x^2 + x + 1)$ **f)** $(x - 4)(x - 1)(x + 2)(x + 3)$

1. Solve.

a) $x(x + 2)(x - 5) = 0$

b) $(x - 1)(x - 4)(x + 3) = 0$

c) $(3x + 2)(x + 9)(x - 2) = 0$

d) $(x - 7)(3x + 2)(x + 1) = 0$

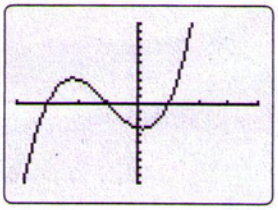
e) $(4x - 1)(2x - 3)(x + 8) = 0$

f) $(2x - 5)(2x + 5)(x - 7) = 0$

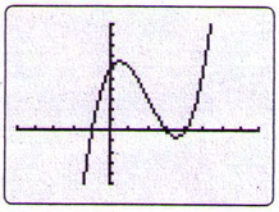
g) $(5x - 8)(x + 3)(2x - 1) = 0$

2. Use the graph to determine the roots of the corresponding polynomial equation. The roots are all integral values.

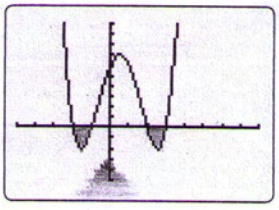
a) Window variables: $x \in [-4, 4]$, $y \in [-10, 10]$



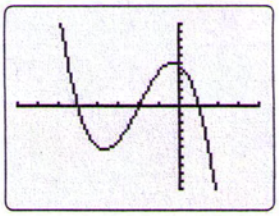
b) Window variables: $x \in [-5, 8]$, $y \in [-10, 20]$, Yscl = 2



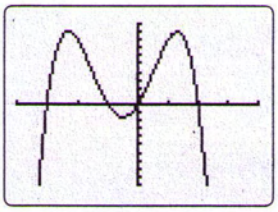
c) Window variables: $x \in [-5, 8]$, $y \in [-10, 20]$, Yscl = 2



d) Window variables: $x \in [-8, 4]$, $y \in [-20, 20]$, Yscl = 2



e) Window variables: $x \in [-4, 4]$, $y \in [-10, 10]$



3. Determine the real roots of each polynomial equation.

a) $(x^2 + 1)(x - 4) = 0$

b) $(x^2 - 1)(x^2 + 4) = 0$

c) $(3x^2 + 27)(x^2 - 16) = 0$

d) $(x^4 - 1)(x^2 - 25) = 0$

e) $(4x^2 - 9)(x^2 + 16) = 0$

f) $(x^2 + 7x + 12)(x^2 - 49) = 0$

g) $(2x^2 + 5x - 3)(4x^2 - 100) = 0$

4. Determine the x-intercepts of the graph of each polynomial function.

a) $y = x^3 - 4x^2 - 45x$

b) $f(x) = x^4 - 81x^2$

c) $P(x) = 6x^3 - 5x^2 - 4x$

d) $h(x) = x^3 + x^2 - 4x - 4$

e) $g(x) = x^4 - 16$

f) $k(x) = x^4 - 2x^3 - x^2 + 2x$

g) $t(x) = x^4 - 29x^2 + 100$

5. Is each statement true or false? If the statement is false, reword it to make it true.

a) If the graph of a quartic function has two x-intercepts, then the corresponding quartic equation has four real roots.

b) All the roots of a polynomial equation correspond to the x-intercepts of the graph of the corresponding polynomial function.

c) A polynomial equation of degree three must have at least one real root.

d) All polynomial equations can be solved algebraically.

e) All polynomial equations can be solved graphically.

6. Solve by factoring.

a) $x^3 - 4x^2 - 3x + 18 = 0$

b) $x^3 - 4x^2 - 7x + 10 = 0$

c) $x^3 - 5x^2 + 7x - 3 = 0$

d) $x^3 + x^2 - 8x - 12 = 0$

e) $x^3 - 3x^2 - 4x + 12 = 0$

f) $x^3 + 2x^2 - 7x + 4 = 0$

g) $x^3 - 3x^2 + x + 5 = 0$

7. Solve by factoring.

a) $2x^3 + 3x^2 - 5x - 6 = 0$

b) $2x^3 - 11x^2 + 12x + 9 = 0$

c) $9x^3 + 18x^2 - 4x - 8 = 0$

d) $5x^3 - 8x^2 - 27x + 18 = 0$

e) $8x^4 - 64x = 0$

f) $4x^4 - 2x^3 - 16x^2 + 8x = 0$

g) $x^4 - x^3 - 11x^2 + 9x + 18 = 0$

8. Solve by factoring.

a) $x^3 - 5x^2 + 8 = -2x$

b) $x^3 - x^2 = 4x + 6$

c) $2x^3 - 7x^2 + 10x - 5 = 0$

d) $x^4 - x^3 = 2x + 4$

e) $x^4 + 13x^2 = -36$

17. Solve. Round answers to one decimal place if necessary.

a) $2(x - 1)^3 = 16$

b) $2(x^2 - 4x)^2 - 5(x^2 - 4x) = 3$

Solutions

Polynomial Equations,

1. a) $x = 0$ or $x = -2$ or $x = 5$
b) $x = 1$ or $x = 4$ or $x = -3$
c) $x = -\frac{2}{3}$ or $x = -9$ or $x = 2$
d) $x = 7$ or $x = -\frac{2}{3}$ or $x = -1$
e) $x = 0.25$ or $x = 1.5$ or $x = -8$
f) $x = 2.5$ or $x = -2.5$ or $x = 7$
g) $x = 1.6$ or $x = -3$ or $x = 0.5$
2. a) $x = -3$ or $x = -1$ or $x = 1$
b) $x = -1$ or $x = 3$ or $x = 4$
c) $x = -2$ or $x = -1$ or $x = 2$ or $x = 3$
d) $x = -5$ or $x = -2$ or $x = 1$
e) $x = -3$ or $x = -1$ or $x = 0$ or $x = 2$
3. a) $x = 4$ **b)** $x = 1$ or $x = -1$ **c)** $x = 4$ or $x = -4$
d) $x = -1$ or $x = 1$ or $x = 5$ or $x = -5$
e) $x = 1.5$ or $x = -1.5$
f) $x = 7$ or $x = -7$ or $x = -3$ or $x = -4$
g) $x = -3$ or $x = 0.5$ or $x = 5$ or $x = -5$
4. a) $-5, 0, 9$ **b)** $-9, 0, 9$ **c)** $-\frac{1}{2}, 0, \frac{4}{3}$
d) $-2, -1, 2$ **e)** $-2, 2$ **f)** $-1, 0, 1, 2$ **g)** $-5, -2, 2, 5$
5. Answers may vary. Sample answers:
a) False. If the graph of a quartic function has four x-intercepts then the corresponding quartic equation has four real roots.
b) True. **c)** False. A polynomial equation of degree 3 has three or fewer real roots. **d)** False. If a polynomial equation is not factorable, the roots can be determined by graphing. **e)** True
6. a) $x = -2$ or $x = 3$ **b)** $x = 5$ or $x = -2$ or $x = 1$
c) $x = 1$ or $x = 3$ **d)** $x = -2$ or $x = 3$
e) $x = -2$ or $x = 2$ or $x = 3$ **f)** $x = -4$ or $x = 1$ **g)** $x = -1$
7. a) $x = -2$ or $x = -1$ or $x = 1.5$ **b)** $x = -0.5$ or $x = 3$
c) $x = -2$ or $x = -\frac{2}{3}$ or $x = \frac{2}{3}$
d) $x = -2$ or $x = 0.6$ or $x = 3$ **e)** $x = 0$ or $x = 2$
f) $x = -2$ or $x = 0$ or $x = 0.5$ or $x = 2$
g) $x = -3$ or $x = -1$ or $x = 2$ or $x = 3$
8. a) $x = -1$ or $x = 2$ or $x = 4$ **b)** $x = 3$
c) $x = 1$ **d)** $x = -1$ or $x = 2$ **e)** no real roots

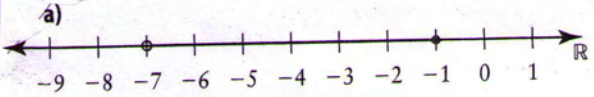
17. a) $x = 3$ **b)** $x \doteq -0.6$ or $x \doteq 0.1$ or $x \doteq 3.9$ or $x \doteq 4.6$

5.

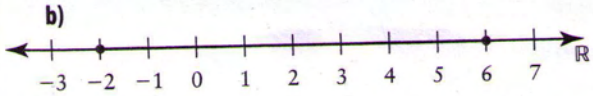
Solve Inequalities

1. Write inequalities for the values of x shown.

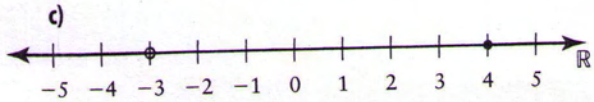
a)



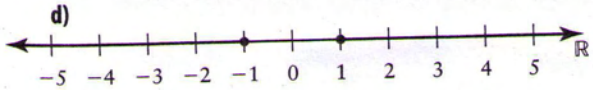
b)



c)



d)



2. Write the intervals into which the x-axis is divided by each set of x-intercepts of a polynomial function.

a) $-1, 5$ **b)** $-7, 2, 0$

c) $-6, 0, 1$ **d)** $-4, -2, \frac{2}{5}, 4.3$

3. Sketch a graph of a cubic polynomial function $y = f(x)$ such that $f(x) < 0$ when $-4 < x < 3$ or $x > 7$ and $f(x) > 0$ when $x < -4$ or $3 < x < 7$.

4. Describe what the solution to each inequality indicates about the graph of $y = f(x)$.

a) $f(x) < 0$ when $-2 < x < 1$ or $x > 6$

b) $f(x) \geq 0$ when $x \leq -3.6$ or $0 \leq x \leq 4.7$ or $x \geq 7.2$

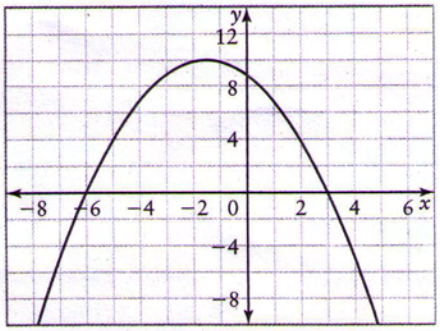
5. For each graph, write

i) the x-intercepts

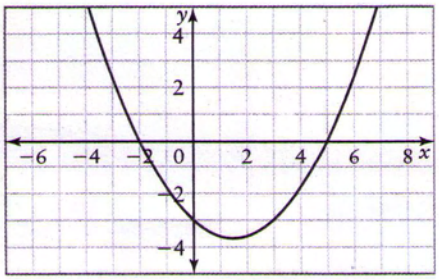
ii) the intervals of x for which the graph is positive

iii) the intervals of x for which the graph is negative

a)



b)



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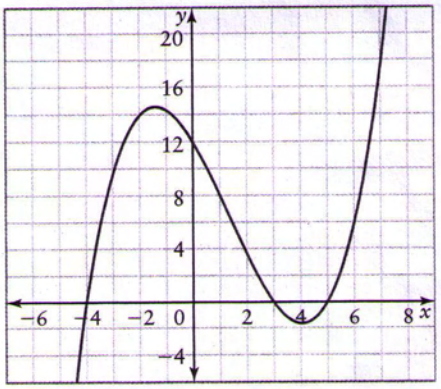
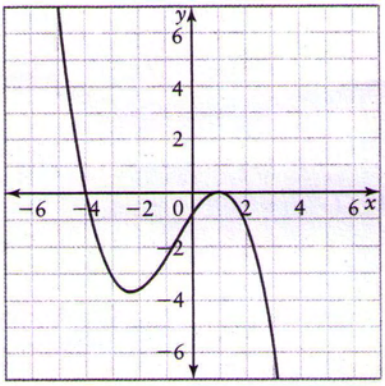


d)



6. Solve each polynomial inequality by graphing the polynomial function.

a) $x^2 - x - 12 < 0$

b) $x^2 + 8x + 15 \leq 0$

c) $x^3 - 6x^2 + 11x - 6 > 0$

d) $x^3 + 8x^2 + 19x + 12 \geq 0$

e) $x^3 - 2x^2 - 9x + 18 < 0$

f) $x^3 + x^2 - 16x - 16 \leq 0$

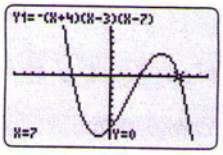
SOLUTIONS

Solving Inequalities

1. a) $-7 < x \leq -1$ **b)** $-2 < x \leq 6$ **c)** $x < -3, x \geq 4$
d) $x \leq -1, x \geq 1$

2. a) $x < -1, -1 < x < 5, x > 5$
b) $x < -7, -7 < x < 0, 0 < x < 2, x > 2$
c) $x < -6, -6 < x < 0, 0 < x < 1, x > 1$
d) $x < -4, -4 < x < -2, -2 < x < \frac{2}{5}, \frac{2}{5} < x < 4.3, x > 4.3$

3.



4. a) $f(x) > 0$ when $x < -2$ or $1 < x < 6$
b) $f(x) < 0$ when $-3.6 < x < 0$ or $x > 4.7$

6. a) $-3 < x < 4$ **b)** $-5 \leq x \leq -3$
c) $1 < x < 2, x > 3$ **d)** $-4 \leq x \leq -3, x \geq -1$
e) $x < -3, 2 < x < 3$ **f)** $x \leq -4, -1 \leq x \leq 4$

5. a) i) $-6, 3$ **ii)** $-6 < x < 3$ **iii)** $x < -6, x > 3$
b) i) $-2, 5$ **ii)** $x < -2, x > 5$ **iii)** $-2 < x < 5$
c) i) $-4, 3, 5$ **ii)** $-4 < x < 3, x > 5$ **iii)** $x < -4, 3 < x < 5$
d) i) $-4, 1$ **ii)** $x < -4$ **iii)** $-4 < x < 1, x > 1$

6. Solve Factorable Polynomial Inequalities Algebraically

1. Solve each inequality. Show each solution on a number line.

a) $x + 3 \leq 5$

b) $2x + 1 > -4$

c) $5 - 3x \geq 6$

d) $7x < 4 + 3x$

e) $2 - 4x > 5x + 20$

f) $2(1 - x) \leq x - 8$

2. Solve by considering all cases. Show each solution on a number line.

a) $(x + 2)(x - 4) > 0$

b) $(2x + 3)(4 - x) \leq 0$

Solve using intervals. Show each solution on a number line.

a) $(x + 3)(x - 2) > 0$

b) $(x - 6)(x - 9) \leq 0$

c) $(4x + 1)(2 - x) \geq 0$

4. Solve.

a) $(x + 2)(3 - x)(x + 1) < 0$

b) $(-x + 1)(3x - 1)(x + 7) \geq 0$

c) $(7x + 2)(1 - x)(2x + 5) > 0$

d) $(x + 4)(-3x + 1)(x + 2) \leq 0$

5. Solve by considering all cases. Show each solution on a number line.

a) $x^2 - 8x + 15 \geq 0$

b) $x^2 - 2x - 15 < 0$

c) $15x^2 - 14x - 8 \leq 0$

d) $x^3 - 2x^2 - 5x + 6 < 0$

e) $2x^3 + 3x^2 - 2x - 3 \geq 0$

6. Solve using intervals.

a) $x^3 + 6x^2 + 7x + 12 \geq 0$

b) $x^3 + 9x^2 + 26x + 24 < 0$

c) $5x^3 - 12x^2 - 11x + 6 \leq 0$

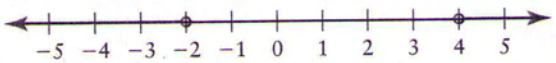
d) $6x^4 - 7x^3 - 4x^2 + 8x + 12 > 0$

Solutions.

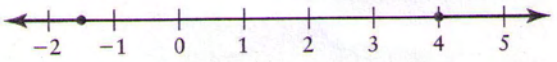
Solving Factorable Polynomial Inequalities Algebraically

1. a) $x \leq 2$ **b)** $x > -\frac{5}{2}$ **c)** $x \leq -\frac{1}{3}$ **d)** $x < 1$ **e)** $x < -2$ **f)** $x \geq \frac{10}{3}$

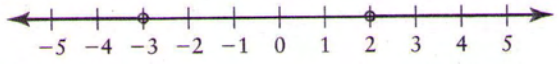
2. a) $x < -2$ or $x > 4$



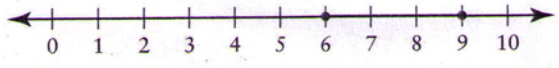
b) $x \leq -\frac{3}{2}$ or $x \geq 4$



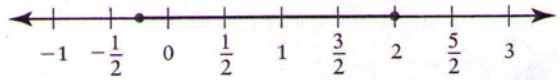
3. a) $x < -3$ or $x > 2$



b) $6 \leq x \leq 9$



c) $-\frac{1}{4} \leq x \leq 2$



4. a) $-2 < x < -1$ or $x > 3$ **b)** $x \leq -7$ or $\frac{1}{3} \leq x \leq 1$

c) $x < -2.5$ or $-\frac{2}{7} < x < 1$ **d)** $-4 \leq x \leq -2$ or $x \geq \frac{1}{3}$

5. a) $x \leq 3$ or $x \geq 5$ **b)** $-3 < x < 5$ **c)** $-\frac{2}{5} \leq x \leq \frac{4}{3}$

d) $x < -2$ or $1 < x < 3$ **e)** $-\frac{3}{2} \leq x \leq -1$ or $x \geq 1$

6. a) approximately $x \geq -5.09$ **b)** $x < -4$ or $-3 \leq x \leq -2$

c) $x \leq -1$ or $\frac{2}{5} \leq x \leq 3$ **d)** true for all intervals

PRACTICE TEST

For questions 1 to 3, select the best answer.

1. Which statement is true for $P(x) = 5x^3 + 4x^2 - 3x + 2$?

A When $P(x)$ is divided by $x + 1$, the remainder is 8.

B $x + 2$ is a factor of $P(x)$.

C $P(-2) = -16$

D $P(x) = (x + 1)(5x^2 - x - 2) - 4$

2. Which of the following is not a factor of $2x^3 - 5x^2 - 9x + 18$?

A $2x - 3$

B $x + 2$

C $x - 2$

D $x - 3$

3. Which set of values for x should be tested to determine the possible zeros of $4x^3 + 5x^2 - 23x - 6$?

A $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6$

B $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm\frac{1}{2}, \pm\frac{2}{3}$

C $\pm 4, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{3}{4}, \pm\frac{1}{4}$

D $\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{3}{4}, \pm\frac{1}{4}$

4. **a)** Divide $x^3 - 4x^2 + 3x - 7$ by $x + 3$. Express the result in quotient form.

b) Identify any restrictions on the variable.

c) Write the corresponding statement that can be used to check the division.

d) Verify your answer.

5. **a)** Determine the value of k such that when $f(x) = x^4 + kx^3 - 2x^2 + 1$ is divided by $x + 2$, the remainder is 5.

b) Determine the remainder when $f(x)$ is divided by $x + 4$.

c) Verify your answer in part b) using long division.

6. Factor.

a) $x^3 - 5x^2 + 2x + 8$

b) $x^3 + 2x^2 - 9x - 18$

c) $x^3 + 5x^2 - 2x - 24$

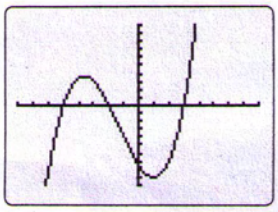
d) $5x^3 + 7x^2 - 8x - 4$

e) $x^3 + 9x^2 + 26x + 24$

f) $2x^4 + 13x^3 + 28x^2 + 23x + 6$

7. Use the graph to determine the roots of the corresponding polynomial equation.

Window variables: $x \in [-8, 8]$, $y \in [-40, 40]$, Yscl = 4



8. Determine the real roots of each equation.

a) $(x^2 + 5)(x - 2) = 0$

b) $(x^2 - 121)(x^2 + 16) = 0$

c) $(x^2 - 2x + 3)(2x^2 - 50) = 0$

d) $(3x^2 - 27)(x^2 - 3x - 10) = 0$

9. Solve by factoring.

a) $x^3 + 4x^2 + 5x + 2 = 0$

b) $x^3 - 13x + 12 = 0$

c) $32x^3 - 48x^2 - 98x + 147 = 0$

d) $45x^4 - 27x^3 - 20x^2 + 12x = 0$

16. Solve by factoring.

a) $9x^2 - 16 < 0$

b) $-x^3 + 6x^2 - 9x > 0$

c) $2x^3 + 5x^2 - 18x - 45 \leq 0$

d) $2x^4 + 5x^3 - 8x^2 - 17x - 6 \geq 0$

Solutions

Chapter 2 Practice Test,

1. C
2. C
3. D
4. a) $\frac{x^3 - 4x^2 + 3x - 7}{x + 3} = x^2 - 7x + 24 - \frac{79}{x + 3}$
b) $x \neq -3$ **c)** $(x + 3)(x^2 - 7x + 24) - 79$
5. a) $k = \frac{1}{2}$ **b)** 193

6. a) $(x - 4)(x - 2)(x + 1)$ **b)** $(x - 3)(x + 2)(x + 3)$
c) $(x - 2)(x + 3)(x + 4)$ **d)** $(x - 1)(x + 2)(5x + 2)$
e) $(x + 2)(x + 3)(x + 4)$ **f)** $(x + 1)(x + 2)(x + 3)(2x + 1)$
7. $x = -5$ or $x = 3$ or $x = -2$
8. a) $x = 2$ **b)** $x = -11$ or $x = 11$ **c)** $x = -5$ or $x = 5$
d) $x = -3$ or $x = 3$ or $x = -2$ or $x = 5$
9. a) $x = -2$ or $x = -1$ **b)** $x = -4$ or $x = 1$ or $x = 3$
c) $x = -1.75$ or $x = 1.5$ or $x = 1.75$
d) $x = -\frac{2}{3}$ or $x = 0$ or $x = \frac{3}{5}$ or $x = \frac{2}{3}$